14.1 14) Evaluate $\int_{-1}^{2}\int_{1}^{2} x\ln y\,dy\,dx$

$$\int_{-1}^{2}\int_{1}^{2} x\ln y\,dy\,dx = \int_{-1}^{2} x\left(y\ln y - y\right)\Big|_{y=1}^{y=2} dx$$

$$= \int_{-1}^{2} x\left(2\ln 2 - 2 - \xcancel{\ln 1}^{\,0} + 1\right) dx$$

$$= \int_{-1}^{2} x\left(2\ln 2 - 1\right) dx$$

$$= \frac{1}{2}x^2\left(2\ln 2 - 1\right)\Big|_{-1}^{2}$$

$$= \frac{2\ln 2 - 1}{2}\left(4 - (-1)^2\right) = \frac{6\ln 2 - 3}{2} = \boxed{3\ln 2 - \frac{3}{2}}$$

14.1 22) Evaluate $\iint_R xye^{xy^2}\,dA$ $\quad R: 0 \le x \le 2,\ 0 \le y \le 1.$

$$= \int_0^2 \int_0^1 xye^{xy^2}\,dy\,dx.$$ First integrate wrt. y.

$$\int_0^1 xye^{xy^2}\,dy = \int_0^x \frac{1}{2}e^u\,du = \frac{1}{2}e^u\Big|_0^x = \frac{1}{2}e^x - \frac{1}{2}.$$

$u(y) = xy^2$

$du = 2xy\,dy.$

$y=0 \Rightarrow u=0$

$y=1 \Rightarrow u=x.$

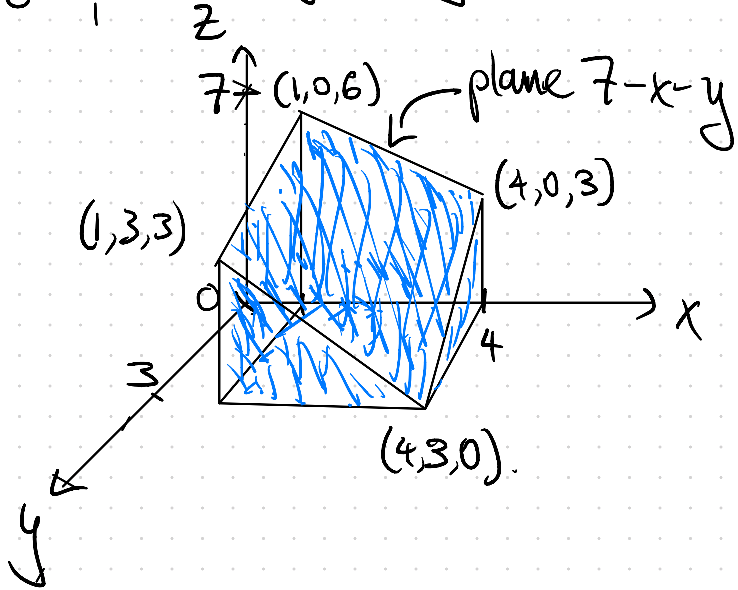
So $$\int_0^2\int_0^1 xye^{xy^2}\,dy\,dx = \int_0^2 \frac{1}{2}e^x - \frac{1}{2}\,dx$$

$$= \frac{1}{2}e^x\Big|_0^2 - \frac{1}{2}x\Big|_0^2$$

$$= \frac{1}{2}e^2 - \frac{1}{2} - 1 = \boxed{\frac{1}{2}e^2 - \frac{3}{2}}$$

14.1 28) Sketch the solid whose volume is given by

$$\int_0^3 \int_1^4 (7-x-y)\,dx\,dy$$



14.1 30) Find the volume of the region bounded above by the elliptical paraboloid $z = 16 - x^2 - y^2$ and below by the square $R: 0 \le x \le 2,\ 0 \le y \le 2$.

$$V = \iint_R z\,dA = \iint_R (16 - x^2 - y^2)\,dA = \int_0^2 \int_0^2 (16 - x^2 - y^2)\,dy\,dx$$

$$= \int_0^2 \left(16y \Big|_{y=0}^{y=2} - x^2 y \Big|_{y=0}^{y=2} - \frac{1}{3}y^3 \Big|_{y=0}^{y=2} \right) dx$$

$$= \int_0^2 \left(32 - 2x^2 - \frac{8}{3} \right) dx = 32x \Big|_0^2 - \frac{2}{3}x^3 \Big|_0^2 - \frac{8}{3}x \Big|_0^2$$

$$= 64 - \frac{16}{3} - \frac{16}{3} = \boxed{\frac{160}{3}}$$

14.1 34) Find the volume of the region bounded above by the surface $z = 4 - y^2$ and below by the rectangle $R: 0 \le x < 1,\ 0 \le y \le 2$.

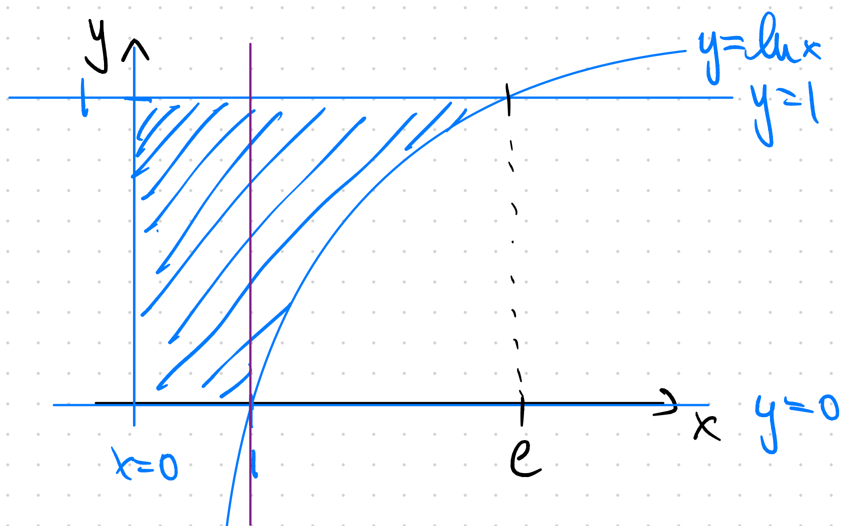
$$V = \iint_R z\,dA = \int_0^1 \int_0^2 (4 - y^2)\,dy\,dx = \int_0^1 \left(4y\Big|_{y=0}^{y=2} - \frac{1}{3}y^3\Big|_{y=0}^{y=2} \right) dx$$

$$= \int_0^1 \left(8 - \frac{8}{3}\right) dx = 8x\Big|_0^1 - \frac{8}{3}x\Big|_0^1 = 8 - \frac{8}{3} = \boxed{\frac{16}{3}}$$

14.2 16) Write an iterated integral for $\iint_R dA$ over the described region R using (a) vertical cross-sections,

(b) horizontal cross-sections

R: Bounded by $y=0$, $x=0$, $y=1$, and $y=\ln x$.



a) $\int_0^1\int_0^1 dy\,dx + \int_1^e\int_{\ln x}^1 dy\,dx$

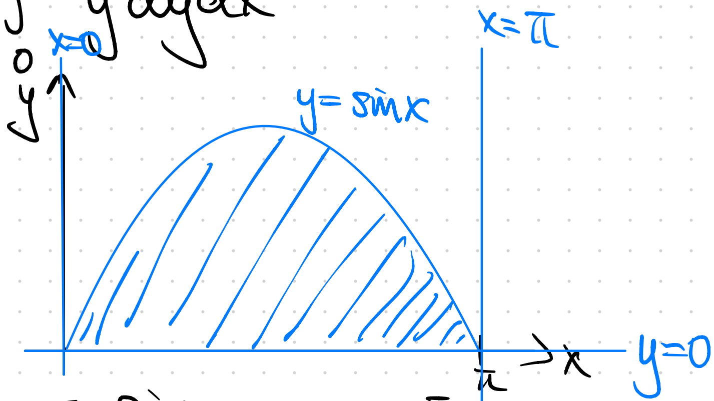
b) $\int_0^1\int_0^{e^y} dx\,dy$

*axes not to scale.

14.2 28) Sketch the region of integration and evaluate the integral

$$\int_0^{\pi}\int_0^{\sin x} y\,dy\,dx$$

Sketch:

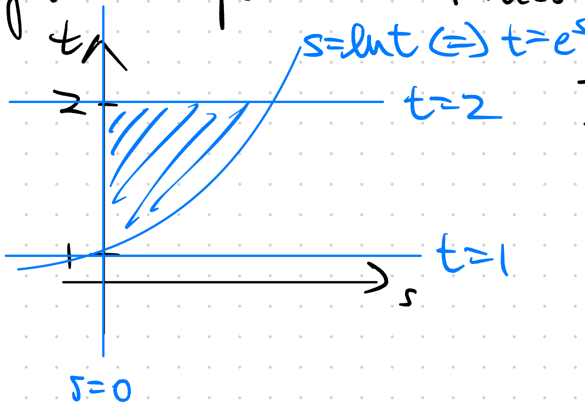


Evaluate: $\displaystyle\int_0^{\pi}\int_0^{\sin x} y\,dy\,dx = \int_0^{\pi} \frac{1}{2}y^2\Big|_{y=0}^{y=\sin x} dx = \int_0^{\pi}\frac{1}{2}\sin^2 x\,dx$

$\displaystyle = \frac{1}{2}\int_0^{\pi}\frac{1-\cos 2x}{2}\,dx = \frac{1}{4}x\Big|_0^{\pi} - \frac{1}{4}\sin 2x\Big|_0^{\pi}$

$= \boxed{\frac{\pi}{4}}$

14.2 36) Integrate $f(s,t) = e^s \ln t$ over the region in the first quadrant of the st-plane that lies above the curve $s = \ln t$ from $t=1$ to $t=2$.



$$I = \int_1^2 \int_0^{\ln t} e^s \ln t \, ds \, dt$$

$$= \int_1^2 e^s \ln t \Big|_{s=0}^{s=\ln t} dt = \int_1^2 (t \ln t - \ln t)\, dt$$

$$= \frac{1}{4} t^2 (2\ln t - 1)\Big|_1^2 - (t\ln t - t)\Big|_1^2$$

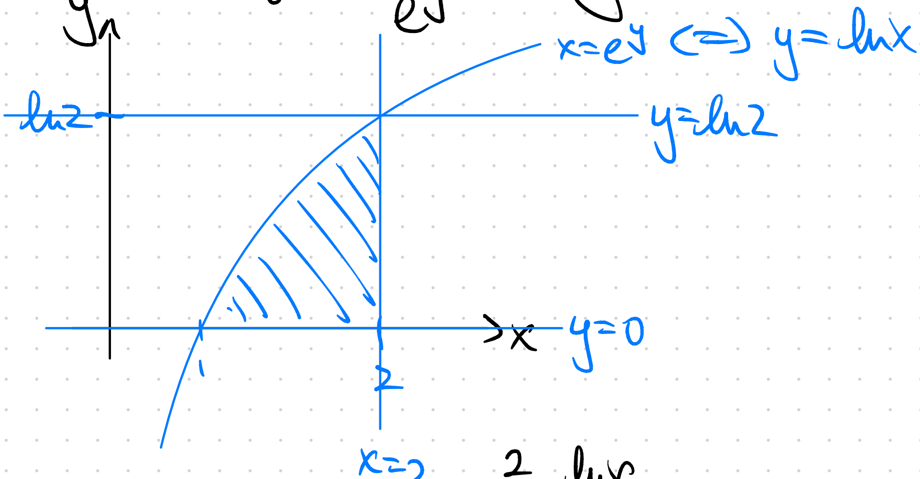
$$= (2\ln 2 - 1) - \left(-\frac{1}{4}\right) - (2\ln 2 - 2) - 1$$

$$= \cancel{2\ln 2} - \cancel{1} + \frac{1}{4} - \cancel{2\ln 2} + \cancel{2} - \cancel{1}$$

$$= \boxed{\frac{1}{4}}$$

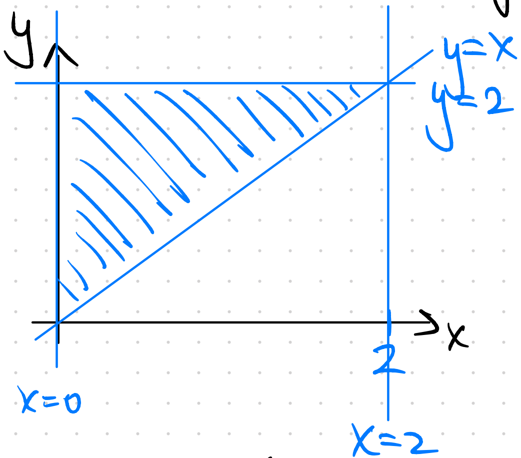
14.2 46) Sketch the region and write an equivalent double integral with the order of integration reversed

$$\int_0^{\ln 2}\int_{e^y}^{2} dx\,dy$$



Reversed integral $= \displaystyle\int_1^2\int_0^{\ln x} dy\,dx$

14.2 56) Sketch the region of integration, reverse the order of integration, and evaluate the integral: $\int_0^2 \int_x^2 2y^2 \sin xy \, dy \, dx$



$$I = \int_0^2 \int_x^2 2y^2 \sin xy \, dy \, dx = \int_0^2 \int_0^y 2y^2 \sin xy \, dx \, dy$$

$$= \int_0^2 -2y \cos xy \Big|_{x=0}^{x=y} dy = \int_0^2 \left(-2y\cos y^2 + 2y\right) dy$$

$u = y^2$

$du = 2y$

$$= \int_0^4 (1 - \cos u)\, du = u\Big|_0^4 - \sin u\Big|_0^4 = \boxed{4 - \sin 4}$$

When $y=0$, $u=0$

$y=2$, $u=4$